Proof

18 marks

1. (a) Write down an expression, in terms of n, for the nth multiple of 5.

.....

(1)

(b) Hence or otherwise

(i) prove that the sum of two consecutive multiples of 5 is always an odd number,

(ii) prove that the product of two consecutive multiples of 5 is always an even number.

(5)
(Total 6 marks)

2. Prove algebraically that the sum of the squares of any two consecutive even integers is never a multiple of 8.

(Total 4 marks)

3. Prove that,

$$(n + 1)^2 - (n - 1)^2$$

is a multiple of 4, for all positive integer values of n.

(Total 3 marks)

4. John says "For all prime numbers, n, the value of $n^2 + 3$ is always an even number".
Give an example to show that John is **not** correct.

(Total 2 marks)

5. Prove algebraically that the sum of the squares of any two odd numbers leaves a remainder of 2 when divided by 4.

(Total 3 marks)